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Solitons in The Domain Structure of an Easy-Axis Ferromagnet

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Abstract. We obtain new solutions of the Landau-Lifshitz model by the inverse scattering technique. They describe solitons strongly associated with the domain structure of an easy-axis ferromagnet. Solitons serve as elementary carriers of macroscopic shifts of the structure and are, under certain conditions, nuclei of the magnetic reversal of a material. We show, that the inhomogeneous elliptic precession of magnetization in a soliton core leads to oscillations of the neighbouring domain walls of the structure. We study the behavior of immobile solitons in the domain structure, which are more simple to observe in experiment. We investigate the structure of solitons near the boundaries of their existence.

1. Introduction

In the bulk samples of ferromagnets with a strong «easy-axis» anisotropy an equilibrium state of the medium represents a stripe domain structure. Investigation of solitons against such strongly nonlinear and inhomogeneous background is a quite difficult problem, usually requiring some simplifications of the Landau – Lifshitz equations. The domain walls are considered to be infinitely thin and plane. Interactions of the discrete domain walls at small deviations from equilibrium states are described by an effective potential. In a such approach self-localized waves of the longitudinal deformations of the domain structure were studied [1], taking into account a nonlinear interaction between the neighbouring domain boundaries. The solitons obtained can be only moving: their velocities exceed the limiting Walker velocity of the single domain wall. The domain structure as if get out of such modulations.

We suppose, that at a theoretical description of quasi-one-dimensional solitons in the stripe domain structure of an «easy-axis» ferromagnet in the first approximation we can neglect the magnetostatic forces in the Landau – Lifshitz equation, since their basic contribution is accounted for by fixing the period of the domain structure. This approximation is better to describe the structure with Bloch domain boundaries, which do not induce the magnetostatic fields. At such approximation the Landau – Lifshitz equation has the form:

$$\partial_t \mathbf{S} = [\mathbf{S}\times\partial_x^2\mathbf{S}] + (\mathbf{n}\cdot\mathbf{S})[\mathbf{S}\times\mathbf{n}], \quad \mathbf{S}^2 = 1, \tag{1}$$

where $\mathbf{S}(x,t)$ is the magnetization, $\mathbf{n} = (0,0,1)$ sets an easy-axis anisotropy; x , t are spatial coordinate and time. Here and below, to simplify formulas we use dimensionless variables.

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The model (1) is integrable. As the result, nonlinear waves and solitons against the homogeneous ground state of an easy-axis ferromagnet were studied in detail [2, 3]. At the same time, solitons in the stripe domain structure are still difficult to describe because of the essentially nonlinear and inhomogeneous ground state of the medium. In the work [4] we have used a special variant of the inverse scattering problem technique to construct new exact solutions of the model (1). We consider the soliton solutions of the model (1) in the stripe domain structure, corresponding to the boundary conditions:

$$\begin{aligned} &\mathbf{S}(x,t) \to \mathbf{S}_2^{(0)} = (\sin\theta_2 \cos\varphi_0,\, \sin\theta_2 \sin\varphi_0,\, \cos\theta_2)\,, \qquad \chi \to +\infty, \\ &\mathbf{S}(x,t) \to \mathbf{S}_1^{(0)} = (\sin\theta_1 \cos\varphi_0,\, \sin\theta_1 \sin\varphi_0,\, \cos\theta_1)\,, \qquad \chi \to -\infty, \end{aligned} \tag{2}$$

where $\theta_j = \pi/2 - \mathrm{am}(\chi + \Delta_j, k)$; $j = 1,2$; $\Delta_1 = \Delta$, $\Delta_2 = 0$, $\chi = x/k$; $\mathrm{am}(\chi, k)$ – is the Jacobi elliptic function with the modulus $0 \le k \le 1$ [5]. The structure shift Δ determine the parameters of the soliton. The angle φ_0 sets the rotation of magnetization (for the Bloch domain walls : $\varphi_0 = \pm\pi/2$).

The domain structure (2) has the period $4Kk$, where $K = K(k)$ is the complete elliptic integral of the first kind. It represents a sequence of the domains of the length $L_0 = 2Kk$, divided by the domain boundaries of the width $l_0 = 2K'k/\pi$, where $k' = \sqrt{1-k^2}$ – is the complementary modulus, $K' = K(k')$. In the bulk samples : $L_0 / l_0 >> 1$ [6]. It is possible only at $k' << 1$. We choose $L_0 / l_0 \approx 9.5$ ($k = 0.9994$, $K \approx 4.75$, $K' \approx \pi/2$).

We have shown, that the solitons can be both mobile and immobile. As well as dislocations in crystals, such solitons can serve as elementary carriers of the macroscopic shifts of the domain structure. However, unlike the case of dislocation in crystals, an elementary shift of the domain structure, accompanied the formation and motion of the soliton, does not depend on the period of the stripe structure, but is determined solely by the construction of the soliton core. The pulsations of the soliton core (due to inhomogeneous precession of magnetization within the core), induce the reciprocal oscillations of the domain walls in the stripe structure, shifted by the core. Below we will discuss the peculiarities of such solitons.

2. Soliton in the stripe domain structure

The soliton in the domain structure has the form [4,7-8]:

$$S_3 = \frac{(|\alpha|^2 - |\beta|^2)\tilde{s} + (\alpha^*\beta + \beta^*\alpha)\tilde{c}}{|\alpha|^2 + |\beta|^2}, \qquad S_1 - \mathrm{i}\, S_2 = \frac{(\alpha^2 - \beta^2)\tilde{c} - 2\alpha\beta\tilde{s}}{|\alpha|^2 + |\beta|^2}\, e^{-\mathrm{i}\varphi_0}\,, \tag{3}$$

where

$$\alpha = c_\mu |m_2|^2 + c_\mu^* |m_1|^2,\ \beta = -\mathrm{i}\, m_1 m_2^* (s_\mu + s_\mu^*), \quad m_1 = a_- e^{y+\mathrm{i}\gamma} + b_+ e^{-y-\mathrm{i}\gamma},\ m_2 = b_- e^{y+\mathrm{i}\gamma} + a_+ e^{-y-\mathrm{i}\gamma},$$

$$a_\pm = m(\tilde{\chi}) \frac{\sigma(\tilde{\chi} + K + \mathrm{i}K' \pm \mu)}{\sigma(K + \mathrm{i}K' \pm \mu)} \exp\left[\mp \frac{\eta_1 \mu \tilde{\chi}}{2K}\right], \quad b_\pm = m(\tilde{\chi}) \frac{\sigma(\tilde{\chi} - K + \mathrm{i}K' \pm \mu)}{\sigma(K + \mathrm{i}K' \pm \mu)} \exp\left[\mp \frac{\eta_1 \mu \tilde{\chi}}{2K} + \eta_1(\tilde{\chi} + \mathrm{i}K' \pm \mu)\right],$$

$$m(\chi) = \left[\frac{\sigma(\mathrm{i}K')\sigma(\mathrm{i}K' + 2K)}{2\sigma(\chi + \mathrm{i}K')\,\sigma(\chi + \mathrm{i}K' + 2K)}\right]^{1/2}; \ y = \mathrm{Re}\left[A(\mu,\chi,t) - \frac{\eta_1 \mu \Delta}{4K}\right] + y_0\,,\ \gamma = \mathrm{Im}\left[A(\mu,\chi,t) - \frac{\eta_1 \mu \Delta}{4K}\right] + \gamma_0\,.$$

Here we use denotations $s_\mu = \mathrm{sn}\,\mu$, $c_\mu = \mathrm{cn}\,\mu$, $\tilde{\chi} = \chi + \Delta/2$, $\Delta = -4\,\mathrm{Re}\,\mu$; y_0, γ_0 – are arbitrary real constants; $\eta_1 = \zeta(2K)$, $\sigma(\chi)$ and $\zeta(\chi)$ are the Weierstrass sigma- and zeta-functions [5] with the periods [$4K, 2\mathrm{i}K'$].

The fine structure and properties of the soliton are determined by the complex parameter $\mu = -\rho + \mathrm{i}\theta$ ($0 < \rho < K$, $-K' \le \theta \le K'$) (see figure 1). The value $\Delta = 4\rho$ represents the macroscopic shift of the stripe domains (in terms of the variable $\chi = x/k$) due to the soliton. The shift is associated with the magnetization reversal of the medium in the soliton core because of displacements of the domain boundaries and magnetization rotations in a small group of neighboring domains. The size of the core $\propto k\Delta = 4k\rho$ is independent on the period $4K(k)k$ of the domain structure.

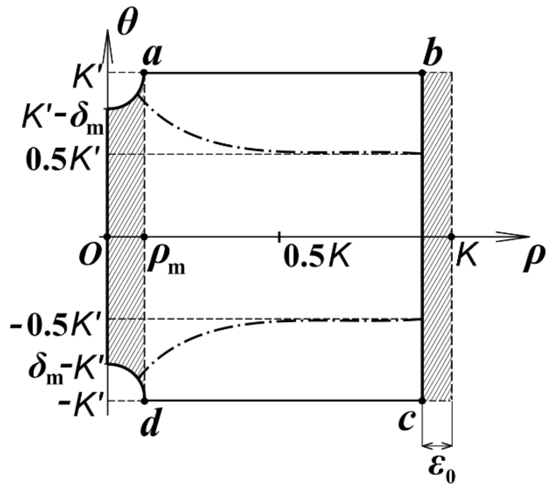

Figure 1. The plane of parameters ρ, θ for the soliton (3) in the domain structure.

The dynamics of the soliton is characterized by the velocity V of the translation of its core and the frequency ω of the precession of magnetization in the core:

$$A(\mu,\chi,t) = [-l^{-1}(x - V\,t) - \mathrm{i}(\eta\, x - \omega t)]/2,$$
$$l = -k\,[\mathrm{Re}\,Z(\mu)]^{-1} > 0,\ \ \eta = -k^{-1}\,\mathrm{Im}\,Z(\mu),\ \ V = -l\,k^{-1}\,\mathrm{Im}(\mathrm{cn}\mu\ \mathrm{dn}\mu),\ \ \omega = k^{-1}\,\mathrm{Re}(\mathrm{cn}\mu\ \mathrm{dn}\mu),$$

where $Z(\mu,k)$ is the Jacobi dzeta-function [5]. Here η is the wave number of the magnetization precession in the core of the soliton. The value l determines the size of the regions to the left and to the right of the soliton core, where the core pulsations cause the oscillations of the domain structure. The resulting size of the soliton in the domain structure is $d \propto k\Delta + 2l$. The values l can vary in a wide range: from the width of one domain wall to the length of several domains. The maximal values of l are obtained at $\rho \to 0$ and $\rho \to K$.

Let us denote nontrivial connection of extended solitons with linear modes of the domain structure. The values of the parameter $\mu = \mathrm{i}\theta$, $-K' < \theta < K'$ correspond to the intra-domain modes [6] with real frequencies:

$$k^{-1} < \omega_1 = \frac{\mathrm{dn}(\theta,k')}{k\,\mathrm{cn}^2(\theta,k')} < \infty\,. \tag{4}$$

At the values $\mu = \mathrm{i}\theta - \rho$ ($0 < \rho = \rho_{\mathrm{m}} = 0.1K$) inside narrow strip to the left from the line *ad* in the figure 1 the soliton (3) transforms into the "cutted" small-amplitude wave of the length $\propto \rho^{-1}$.

The values $\mu = -K + \mathrm{i}\theta$, $-K' < \theta < K'$ correspond to the imaginary frequencies of the second branch of the linear modes:

$$\omega_2 = \frac{\mathrm{i}\,k'^2\,\mathrm{sn}(\theta,k')\,\mathrm{cn}(\theta,k')}{k\,\mathrm{dn}^2(\theta,k')}\,.$$

This mode has the negligibly small increment of increase over time: $|\omega_2|_{\max} = (1-k)/k^{3/2} \approx 6\cdot 10^{-4} << 1$ at $L_0/l_0 \approx 9.5$. In such a situation, the growth of longitudinal deformations of a domain structure is restrained by the law of conservation of the *Oz*-projection of magnetization:

$$\int_0^R S_3(x,\ t)\ \mathrm{d}x = \ \mathrm{const}$$

(where R is the size of the sample) and by the nonlinear interactions that are not considered in the linear analysis of stability of the domain structure [7]. In this work we show, that the nonlinear interactions lead to localization of perturbations and appearance of the particle-like solitons, strongly connected with the domain structure. In the small interval $K-\varepsilon_0 \approx 0.9K < \rho < K$ to the right from the line *bc* in the figure 1 the solution (3) is an analogue of an extended shear soliton within the anharmonic chain of the domain walls [1].

In the general case, the direction of the soliton velocity is set by the sign of the parameter θ: $V = -\mathrm{sign}\theta$. The soliton is immobile at $\theta = 0, \pm K'$ ($\mu \in ab$, $\mu \in cd$ on the figure. 1). When the parameter $|\theta|$ grows from zero to K', the absolute value of the velocity begin to increase, approaching the maximum value $V_{\max}(\rho)$, and then decreases up to zero. The values of ρ and θ, corresponding to the maximal soliton velocity, are shown by two hatched-dotted lines in the figure 1. The value $V_{\max}$ essentially depends on ρ, that is, on the size of the soliton core (see figure. 2). The velocity $V_{\max}$ is small when the size of the soliton core tends to the length $2Kk$ of the one domain.

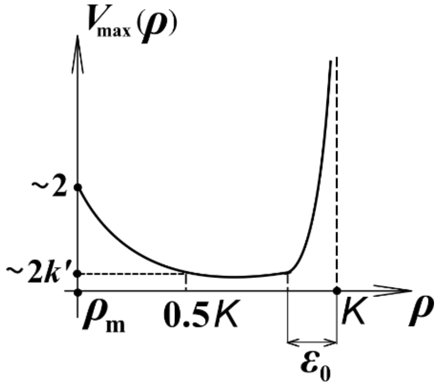

Figure 2. The value $V_{\max}$.

3. Immobile solitons

Immobile solitons are most simple to observe. As an example let us consider the soliton with the parameter $\mu = -\rho$ ($0.4K \le \rho \le 0.6K$) (figure 3), which represents the nucleus of remagnetizing in the center of the one of the domain of the stripe domain structure. Its internal precession frequency: $0 < \omega = k^{-1}\mathrm{cn}\rho\,\mathrm{dn}\rho < k^{-1}$ lies within the energy gap of the spectrum of spin waves (4). Therefore, such solitons are well-generated and diagnosized in the numerical experiments [7, 8]. The solid and hatched lines in the figure 3 correspond to the angle $\Theta = \mathrm{Arc}\cos S_3$ at the moments of time $t = 0$ and $t = T/2$, when $S_3(\tilde{\chi} = K)$ approaches a maximum (T – is the precession period). The noncircular precession in the soliton localization region induces periodic synphase shifts of the domains along the structure on the value $\Delta\chi \le 0.4K$ and the changes of the angle Θ on the value $\Delta\Theta \propto 0.05\,\pi$. In the figure 3 the mutual directions of translations are depicted by the arrows. Outside of the soliton core they are exponentially decreasing. The numbers in circles enumerate peculiar points, in which the shifts of the domains are absent. The position of these points in the structure changes, depending on the values ρ.

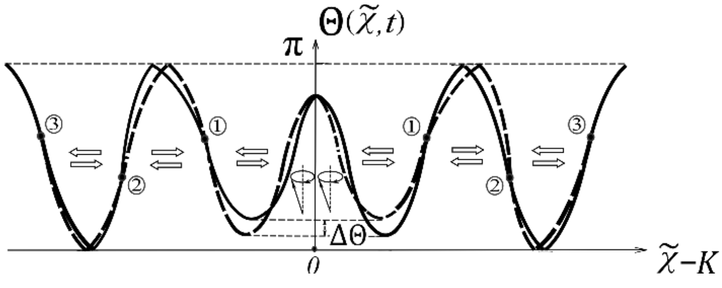

Figure 3. Oscillations of the domain structure near the core of the soliton (3) at $\theta = 0$.

For the immobile solution with $\mu = -\rho \pm \mathrm{i}K'$ ($0.4K \le \rho \le 0.6K$) there are no such points (see figure 4). This soliton represent a nucleus of magnetization reversal inside one of the domains of the structure. The internal precession frequency in the core of the soliton lies within the interval: $0 < |\omega| = \mathrm{cn}\rho\,\mathrm{dn}\rho / (k\,\mathrm{sn}^2\rho) < \infty$. Unlike the previous case, when the center of the soliton lies in the middle of the domain, the magnetization in it always reaches saturation. The phases of the precession of magnetization around *Oz*–axis to the right and to the left from the center of the soliton differ by π. That makes difficult to observe such solitons on the experiment [7, 8].

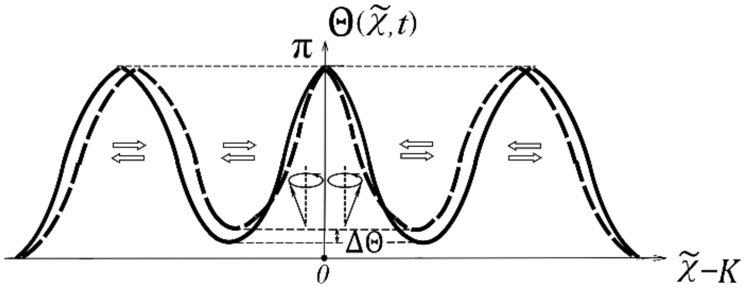

Figure 4. Soliton (3) at $\theta = \pm K'$. The denotations on the figure 4 are the same as on the figure 3.

The results obtained generalize an analysis of immobile solitons against homogeneous ground state of an easy-axis ferromagnet ($k \to 1$), that at first was made in the works of the theoretical group of A.M. Kosevich [2].

4. Solitons near the boundaries of their existence

The behavior of solutions in the regions of instability of the structure in the linear approximation (at $\mu = -K + |\varepsilon| + \mathrm{i}\theta$, $|\varepsilon| << 1$) is of a special interest. An instability leads to extended modulations of the domain structure, which, in general case, move with a large velocity. The cases $\mu = -K + 0$ and $\mu = -K + \mathrm{i}K' + 0$, when the modulations of the structure are immobile, are the most appropriate to observe. At such μ the soliton shifts the domain structure on the period. Therefore, the solution (3) has one and the same asymptotics at $\chi \to +\infty$ and $\chi \to -\infty$: $\mathbf{S} \to \mathbf{S}_2^{(0)}$ (see (2)).

In the limit $\mu = -K + 0$ the excitation (3) looks simpler, when its center is within the middle of the domain wall of the structure. Then the magnetization in the two domains near the center $\chi = 0$ is weakly changing with time. At $-\infty < t < 0$ the expression (3) describes aperiodic translations of the domain walls with even numbers 2*m*, where *m* is integer number, on the value δ_{2m} which is less, than 0.1 period of the structure, to the positions, depicted on the figure 5 by the hatched-dotted lines. Then, at $0 < t < \infty$ the domain walls come to their initial positions.

In the case $\mu = -K + \mathrm{i}K' + 0$ the character of the excitation significantly depends on the position of its center. If the center of excitation is in the middle of the domain wall, then the solution (3) describes complicated process of remagnetizing (see figure 6). At $-\infty < t < 0$ two domains near the center reduce, passing through the positions, depicted by hatched-dotted and hatched lines, whereas two successive domains extend up to the whole period of the structure. At $0 < t < \infty$ there is an opposite process.

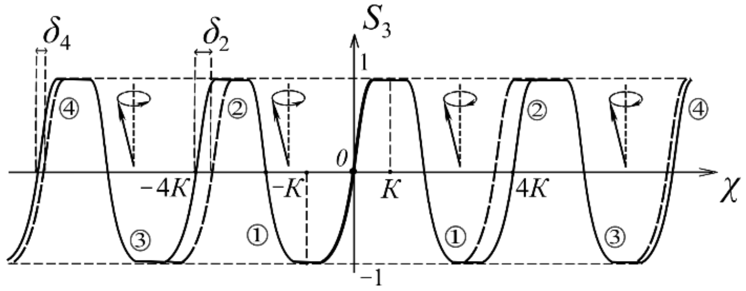

Figure 5. Aperiodic translation of the structure in excitation (3), localized on the immobile domain wall at $\chi = 0$ ($\mu = -K + 0$).

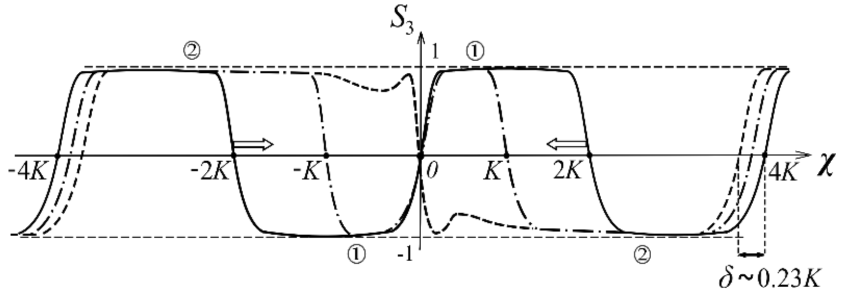

Figure 6. The region of soliton localization (3) with the center in the middle of the domain wall $\chi = 0$ ($\mu = -K + \mathrm{i}K' + 0$).

5. Conclusion

In this work we have discussed the peculiarities of solitons in the domain structure of an easy-axis ferromagnet. These solitons are structurally stable and serve as elementary carries of macroscopic shifts of the structure. A strong interaction of solitons with the domain structure should be taken into account, when analyzing the processes of magnetization reversal of the material. It is also can be important for the development of new devices for storing and processing the information.

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